SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART I – TRIGONOMETRY REVIEW)

Department of Mathematics and Statistics, University of Houston-Downtown

1. Find the other five trigonometric functions of θ using the given information.

 $\tan \theta = \frac{5}{12}; \sin \theta > 0$

- 2. How far are you from the base of a tree 100 feet tall if the angle of elevation to the top of the tree is 30°?
- 3. Graph exactly one period of each of the following functions.

A. $y = 2 \sin x + 1$

B. $y = \tan x$

C. $y = \cos 2x - 2$

4. Prove the following identities by converting one side into the other.

A. $\sec x + \tan x = \frac{\csc x + 1}{\csc x \cos x}$ B. $2 - \sin^2 \theta = 1 + \cos^2 \theta$

C. $\frac{1}{1 + \cos y} + \frac{1}{1 - \cos y} = 2 \csc^2 y$

5. Find the exact value of each of the following expressions.

A. $Arc \sin \frac{1}{2}$

B. $\cos^{-1} \frac{\sqrt{2}}{2}$

B. $\cos^{-1} \frac{\sqrt{2}}{2}$ C. $\cos^{2} 15^{\circ} + \sin^{2} 15^{\circ}$ E. $\tan[\sec^{-1}(-3)]$ F. $\cot \frac{\pi}{4}$

D. csc 60°

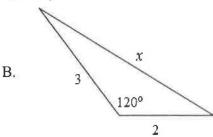
6. Solve the following equations for all x such that $0 \le x < 2\pi$.

A. $2\cos x \sin x = \sin x$

B. $\sec^2 x - \sec x - 2 = 0$

C. $\cos^2 x = \frac{1}{2}$

7. Solve the following triangles for the unknown quantity.



8. For the polar equation $r = 2 - 3 \sin \theta$, find the missing coordinate of the following points so that the resulting point will be in the graph of this equation.

A. $\left(?, \frac{5\pi}{4}\right)$

B. (2,?)

C. $\left(?, \frac{3\pi}{2}\right)$

9. Change the polar coordinates $\left(-3, \frac{5\pi}{4}\right)$ to rectangular coordinates.

SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART II - PRECALCULUS REVIEW)

Department of Mathematics and Statistics, University of Houston-Downtown

- 1. Find an equation of the line that passes through the point (1, -2) and that is perpendicular to the line 2x + 4y = 1.
- 2. Find the inverse function of $f(x) = \frac{2x-5}{x-3}$, if it exists.
- 3. Find all three zeros (or roots) of the polynomial $P(x) = x^3 + x$.
- 4. Solve for x: $\log(28 + x) = \log(2 x) + \log(2 x)$.
- 5. Solve for x: $7^{3x+1} = 100$.
- 6. Find all horizontal and vertical asymptotes, if any, of $r(x) = \frac{5x+21}{x^2+10x+25}$.
- 7. If $f(x) = 3x^2 + 1$, simplify the following expression: $\frac{f(x+h)-f(x)}{h}$.
- 8. Identify the conic section given by the following equation and find its center:

$$x^2 + y^2 - 4x + 10y + 25 = 0$$

- 9. Find the rectangular coordinates for the point whose polar coordinates are $(\sqrt{2}, -\pi/4)$.
- 10. Find functions f(x) and g(x) so that $f(g(x)) = (2x + 1)^3$.
- 11. A function f is given, and the indicated transformations are applied to its graph, in the given order. Write the equation for the final transformed function g(x).
 - $f(x) = x^2$; shift 2 units to the left and reflect in the x-axis.
- 12. Find the equation of the quadratic function f(x) whose graph has vertex (3,5) and y-intercept 23.
- 13. Find the value of b if $log_3b = -2$.
- 14. Find the sum of the first five terms of the arithmetic sequence whose first term is 25 and common difference is -2.
- 15. Find all possible rational zeros (or roots) of the polynomial $f(x) = 2x^3 7x^2 + 10x 6$.
- 16. Find the formula for a polynomial f of degree 3 with integer coefficients such that f(1) = -2 and both 1 i and 3 are zeros (or roots) of the polynomial.
- 17. Find the value of $tan^{-1}(-1)$.
- 18. Find polar coordinates for the point whose rectangular coordinates are (0, -3).
- 19. Find the sum of the first four terms of the geometric sequence whose first term is 3 and common ratio is 2.
- 20. Find the equation of the circle that has a diameter with endpoints at (-3, -2) and (5, 4).
- 21. Write the complex number $-1 + i\sqrt{3}$ in the polar form $r(\cos\theta + i \cdot \sin\theta)$.
- 22. Use DeMoivre's Theorem to compute $(-1 + i\sqrt{3})^{12}$.
- 23. Find the two foci of the ellipse with equation

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

24. Evaluate the expression

$$\sum_{k=0}^{3} (k^3 + 2k)$$

25. A sequence is defined recursively by $F_n = F_{n-1} + F_{n-2}$. If $F_0 = 0$ and $F_1 = 1$, find F_6 .

In problems 26-31, sketch the graph of the given function or equation.

$$26. \frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$26. \frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$27. f(x) = \frac{2x - 3}{x - 2}$$

$$28. y = e^x + 2$$

28.
$$v = e^x + 2$$

28.
$$y = e^{-x^2} + 2$$

29. $y = \begin{cases} 3, & \text{if } x \le 0 \\ -x^2 - 2, & \text{if } x > 0 \end{cases}$
30. $y = x^3 + 2x^2 - 5x - 3$

$$30. y = x^3 + 2x^2 - 5x - 3$$

31.
$$y = \log_{1/2}(x+3)$$

- 32. A polynomial of degree 4 can have at most how many x-intercepts?
- 33. A polynomial of degree 4 can have at most how many local extrema?

ANSWERS FOR SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART I – TRIGONOMETRY REVIEW)

$$\sin \theta = \frac{5}{13}$$

$$\csc\theta = \frac{13}{5}$$

$$\cos\theta = \frac{12}{13}$$

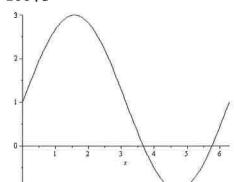
$$\sin \theta = \frac{5}{13}$$
 $\cos \theta = \frac{13}{5}$ $\cos \theta = \frac{12}{13}$ $\sec \theta = \frac{13}{12}$ $\tan \theta = \frac{5}{12}$ $\cot \theta = \frac{12}{5}$

$$\tan\theta = \frac{5}{12}$$

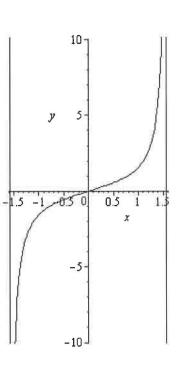
$$\cot \theta = \frac{12}{5}$$

2. Distance =
$$100\sqrt{3}$$

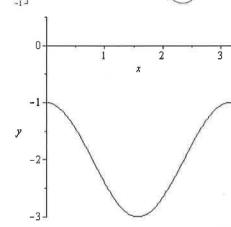
3. A. 3



B.



C.



5. A. $Arc \sin \frac{1}{2} = 30^{\circ} \text{ or } \frac{\pi}{6}$ B. $Cos^{-1} \frac{\sqrt{2}}{2} = 45^{\circ} \text{ or } \frac{\pi}{4}$ C. $cos^2 15^{\circ} + sin^2 15^{\circ} = 1$

B.
$$\cos^{-1}\frac{\sqrt{2}}{2} = 45^{\circ} \text{ or } \frac{\pi}{4}$$

C.
$$\cos^2 15^\circ + \sin^2 15^\circ = 1$$

D.
$$\csc 60^{\circ} = \frac{2\sqrt{3}}{3}$$

E.
$$\tan[\sec^{-1}(-3)] = -2\sqrt{2}$$
 F. $\cot\frac{\pi}{4} = 1$

$$\overline{2}$$
 F. $\cot \frac{\pi}{4} = 1$

6. A. $x = 0^{\circ}, 60^{\circ}, 180^{\circ}, 300^{\circ} \text{ or } x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ B. $x = 60^{\circ}, 180^{\circ}, 300^{\circ} \text{ or } x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ C. $x = 45^{\circ}, 135^{\circ}, 225^{\circ}, 315^{\circ} \text{ or } x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

B.
$$x = 60^{\circ}, 180^{\circ}, 300^{\circ} \text{ or } x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

7. A.
$$x = 5\sqrt{2}$$

B.
$$x = \sqrt{19}$$

8. A.
$$x = \frac{4 + 3\sqrt{2}}{2}$$

B.
$$x = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \cdots$$

9.
$$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$$

ANSWERS FOR SAMPLE PLACEMENT TEST TO BYPASS MATH 1505 (PART II – PRECALCULUS REVIEW)

| 1.2x - y = 4 | $2. \ f^{-1}(x) = \frac{3x-5}{x-2}$ | $3. \ x = 0, x = \pm i$ |
|--|---|---|
| 4. $x = -3$ | $5. x = \frac{2}{3 \log 7} - \frac{1}{3}$ | 6. Horizontal asymptote is $y = 0$. Vertical asymptote is $x = -5$. |
| 7. $6x + 3h$ | 8. A circle with center (2, -5). | 9. (1,-1) |
| 10. $f(x) = x^3$, g(x) = 2x + 1 | 11. $g(x) = -(x+2)^2$ | 12. $f(x) = 2x^2 - 12x + 23$ |
| 13. 1/9 | 14. 105 | 15. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$ |
| $ \begin{array}{c} 16. \\ f(x) = x^3 - 5x^2 + 8x - 6 \end{array} $ | 17. $-\frac{\pi}{4}$ | 18. $(3, \frac{3\pi}{2})$ |
| 19. 45 | 20. $(x-1)^2 + (y-1)^2 = 25$ | 21. $2(\cos\frac{2\pi}{3} + i \cdot \sin\frac{2\pi}{3})$ |
| 22. 2 ¹² | 23. $(\sqrt{11}, 0), (-\sqrt{11}, 0)$ | 24. 48 |
| 25. 8 | 26. | 27. |
| 28. | 29. | 30. |
| 31, | 32. 4 | 33. 3 |