Final exam review – Math 2402

1. Find the derivative of the function.

1a. $f(x) = \ln\left(\frac{2x}{x+3}\right)$ $\boxed{= \frac{3}{x^2+3x}}$

1b. $g(x) = x^2 \ln x$ $\boxed{= 2x \ln x + x}$

1c. $h(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$ $\boxed{= -\frac{4}{x^3+4x}}$

2. Use L'Hopital's rule to find the limit.

2a. $\lim_{x\to\infty} \frac{2x^2+3x+1}{x^2-3}$ $\boxed{= 2}$

2b. $\lim_{x\to\infty} \frac{x^2-9}{e^x}$ $\boxed{= 0}$

2c. $\lim_{x\to 0} \frac{e^{4x}-1-4x}{x^2}$ $\boxed{= 4}$

2d. $\lim_{x\to 0} \frac{\mathrm{x}-\ln(x+1)}{x^2}$ $\boxed{= \frac{1}{2}}$

3. Find the first derivative of the following functions.

3a. $f(x) = x^3 e^x$

$\boxed{f'(x) = 3x^2 e^x + x^3 e^x}$.

3b. $y = e^x(\sin x + \cos x)$

$\boxed{\frac{dy}{dx} = 2e^x \cos x}$.

3c. $y = \frac{e^x+1}{e^x-1}$

$\boxed{y' = \frac{-2e^x}{(e^x-1)^2}}$.

3d. $f(x) = 4^{\pi x^2}$

$\boxed{f'(x) = \ln 4 \; 4^{\pi x} \; 2\pi x}$.

3e. $f(x) = 4^3$

$\boxed{f'(x) = 0}$.

3f. $f(x) = x^x$

$\boxed{f'(x) = x^x \ln x + x^x}$.

4. Find the indefinite integral.

4a. $\int \frac{9}{5-4x} dx$

$= -\frac{9}{4}\ln|5-4x| + c.$

4b. $\int \frac{x^2}{5-x^3} dx$

$= -\frac{1}{3}\ln|5-x^3| + c.$

4c. $\int \frac{(\ln x)^2}{x} dx$

$= \frac{1}{3}(\ln x)^3 + c.$

4d. $\int e^{-x^4} x^3 \, dx$

$= -\frac{1}{4}e^{-x^4} + c.$

4e. $\int 4^x \, dx$

$= \frac{1}{\ln 4}4^x + c.$

5. Find the derivative for the following trigonometric functions.

5a. $g(x) = 7^{\tan x}$

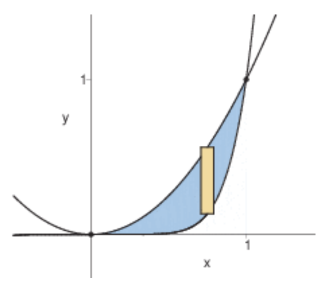
$g'(x) = \ln 7 \; 7^{\tan x} \sec^2 x.$

5b. $f(x) = x^2 \; \arcsin(x)$

$f'(x) = 2x \arcsin x + \frac{x^2}{\sqrt{1-x^2}}.$

6. Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the x-axis.

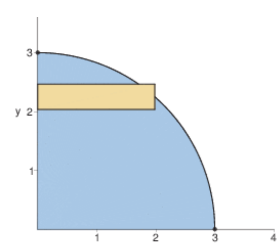
$$y = x^2, \quad y = x^7$$



$\frac{2}{15}\pi$

7. Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the y-axis.

$$y = \sqrt{9 - x^2}$$



18π

8. Find the indefinite integral.

8a. $\int \frac{2}{e^{-x}+1}\ dx$

$\boxed{= 2\ln(e^x + 1) + c}$.

8b. $\int \frac{1}{25+4x^2}\ dx$

$\boxed{= \frac{1}{10}\arctan\frac{2x}{5} + c}$.

8c. $\int x^3 e^x dx$

$\boxed{= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c}$.

8d. $\int x^5 \ln 3x \quad dx$

$\boxed{= \frac{1}{6}x^6 \ln 3x - \frac{1}{36}x^6 + c}$

8e. $\int e^{-3x} \sin 5x \ \ dx$

$\boxed{= \frac{3}{16}\sin 5x \ \ e^{-3x} + \frac{5}{16}\cos 5x \ \ e^{-3x} + c.}$

8f. $\int \arctan x \ \ dx$

$\boxed{= x\arctan x - \frac{1}{2}\ln(x^2 + 1) + c.}$

9. Find an indefinite integral.

9a. $\int \cos^5 x \sin x \ \ dx$

$\boxed{= -\frac{1}{6}\cos^6 x + c}$.

9b. $\int \cos^4 x \ \ dx$

$\boxed{= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c}$.

9d. $\int \sec^4 2x\, dx$

$\boxed{= \frac{1}{2}\tan 2x + \frac{1}{6}\tan^3 2x + c}$.

9e. $\int \tan^3\frac{\pi x}{2}\sec^2\frac{\pi x}{2}\ dx$

$\boxed{= \frac{1}{2\pi}\tan^4\frac{\pi}{2}x + c}$.

9f. $\int \frac{4}{x^2\sqrt{16-x^2}}\,dx$

$= -\frac{\sqrt{16-x^2}}{4x} + c$.

9g. $\int \frac{9x^3}{\sqrt{1+x^2}}\,dx$

$= 3\,(1+x^2)^{\frac{3}{2}} - 9\sqrt{1+x^2} + c$.

9h. $\int e^x\sqrt{1-e^{2x}}\,dx$

$= \frac{1}{2}\arcsin e^x + \frac{1}{2}e^x\sqrt{1-e^{2x}} + c$.

9i. $\int \frac{x^2-6x+2}{x^3+2x^2+x}\,dx$

$= 2\ln|x| - 10\ln|x+1| - \frac{9}{x+1} + c$.

9j. $\int \frac{x^2}{x^4-2x^2-8}\,dx$

$= \frac{1}{6}\ln|x+2| - \frac{1}{6}\ln|x-2| + \frac{\sqrt{2}}{6}\arctan\frac{\sqrt{2}x}{2} + c$.

9k. $\int \frac{e^x}{(e^x-1)(e^x+4)}\,dx$

$= \frac{1}{5}\ln|e^x-1| - \frac{1}{5}\ln(e^x+4) + c.$.

10. Determine whether the integral converges or diverges. If it is convergent find its value.

10a. $\int_e^{\infty} \frac{dx}{x\sqrt{\ln x}}$

The integral diverges

10b. $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$

The integral converges and the integral's value is $2\sqrt{3}$.

11. Confirm that the Integral Test can be applied to the series. Then use the Integral Test to determine the convergence or divergence of the series.

11a. $\sum_{n=1}^{\infty} \frac{1}{2n}$

Series converges

11b. $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$

Series diverges

12. Determine whether each series is a p −series

12a. $\sum_{n=1}^{\infty} \frac{1}{n^{1.4}}$

Yes, it's a p-series

12b. $\sum_{n=1}^{\infty} \frac{1}{n^{-2}}$

No. it's not a p-series

13. Determine the convergence or divergence of series.

13a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

Series diverges

13b. $\sum_{n=1}^{\infty} \frac{3}{n^{\frac{5}{3}}}$

Series converges

13c. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

Series converges

13d. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

Series converges

13e. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$

Series diverges

13f. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

Series converges

13g. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$

Series diverges

14. Find the radius of convergence.

14a. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$

$R = 5$.

14b. $\sum_{n=0}^{\infty} \frac{(4x)^n}{n^2}$

$R = \frac{1}{4}$..

15. Find the Taylor polynomial P_4 for $f(x) = \ln(x)$ centered at $x = 1$.

$$P_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$